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Data assimilation with EKF for FLake: problems and perspectives

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Outlook

- Introduction
- Lake model FLake from DA point of view
- Formulation of EKF
- Study of components of vector H and matrix M
- Results, problems: effect of early spring observations, decoupling
- A posteriori statistics and cross-validation
- Conclusions, perspectives

Introduction

Why data assimilation for lakes is needed?

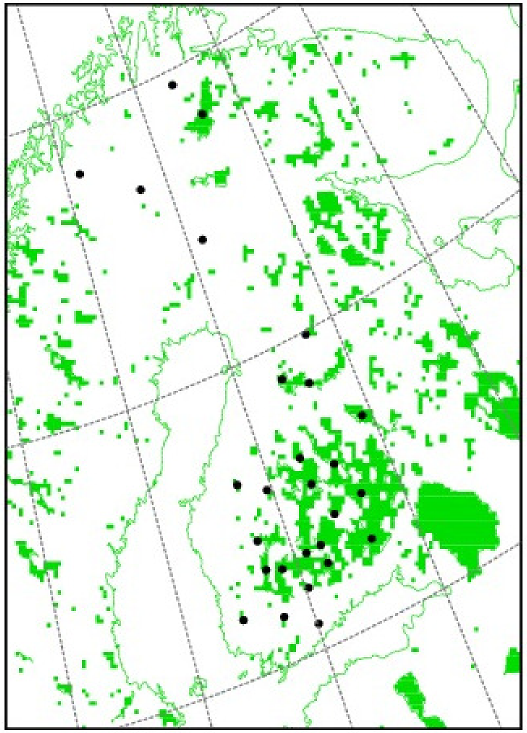
- To combine a lake model (parameterization) with lake observations
- To initialize prognostic variables of a lake model (parameterization)
- To correct model errors, which come from the uncertain initial state
- To get better knowledge about some unmeasured lake characteristics from measured ones (reanalysis)

Data assimilation for:

- Lake model: FLake, 1D aspect
- Lake observations: the lake surface temperature
- Method: Extended Kalman Filter

In-situ (SYKE) observations of LWST:

- 27 lakes in Finland
- Ice-free period
- Daily, at 8.00 EET



In addition, MODIS observations of LWST:

- Early spring period, just after ice break-up
- 4 lakes
- Picked manually, referred to 8.00 EET

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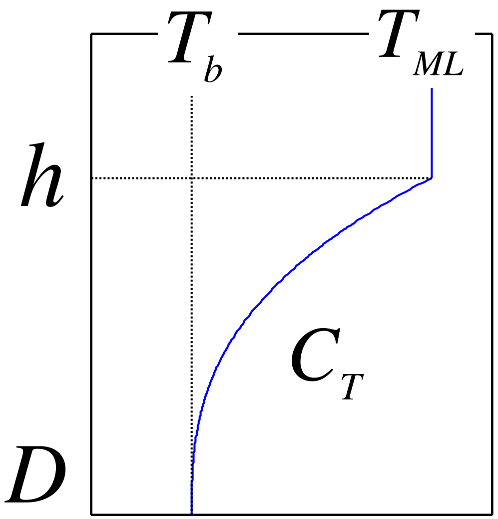
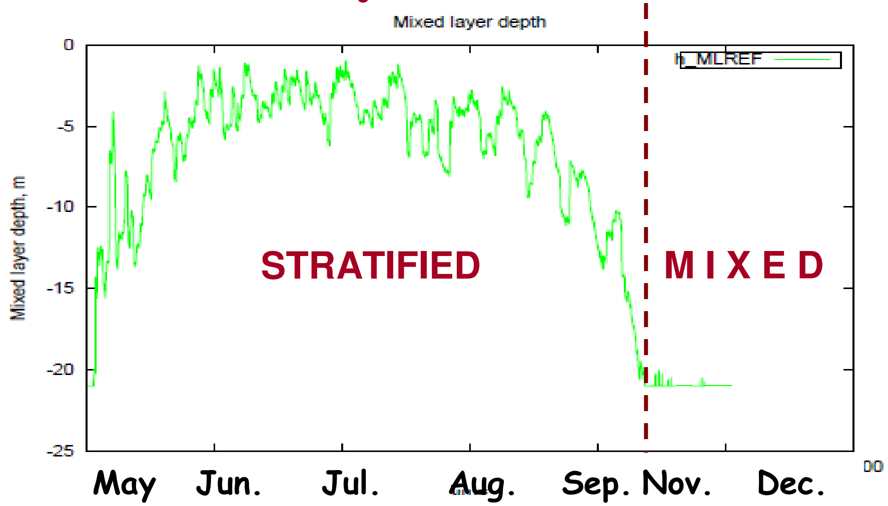
Lake model FLake from DA point of view





Prognostic equations:

- the mean water temperature: $\overline{T}$
- the bottom temperature
- the mixed layer depth
- the shape factor

$$\eta = 1 - \frac{h}{D}$$

Diagnostic equation:

- the mixed layer (surface) temperature

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Formulation of EKF

stratified/mixed regime

State vector

$$\mathbf{X} = \begin{bmatrix} \overline{T} & \eta & T_b & C_T \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{X} = \begin{bmatrix} \overline{T} \end{bmatrix}$$

Obs vector

$$\mathbf{Y} = \begin{bmatrix} T_{ML}^{o} \end{bmatrix}$$

Obs operator $H(\mathbf{X})$

$$T_{ML} = \frac{\overline{T} - C_T \eta T_b}{1 - C_T \eta}$$

$$T_{ML} = \overline{T}$$

Linearised obs operator:

stratified regime

$$\mathbf{H} = \left[\frac{1}{1 - C_T \eta} \quad \frac{C_T (\overline{T} - T_b)}{(1 - C_T \eta)^2} \quad \frac{-C_T \eta}{1 - C_T \eta} \quad \frac{\eta (\overline{T} - T_b)}{(1 - C_T \eta)^2} \right]$$

mixed regime

$$\mathbf{H} = 1$$

Model operator: $M(\mathbf{X})$ - *FLake*

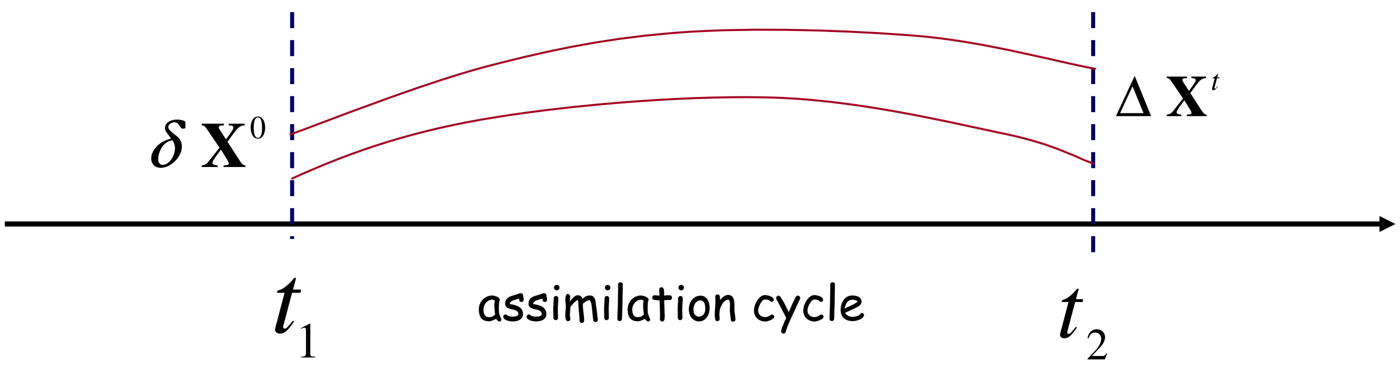
Linearised model operator: $\mathbf{M} = \dfrac{\partial x_i^t}{\partial x_j^0}$

$$\mathbf{M} = \begin{bmatrix} \dfrac{\partial \overline{T}^t}{\partial \overline{T}^0} & \dfrac{\partial \overline{T}^t}{\partial \eta^0} & \dfrac{\partial \overline{T}^t}{\partial T_b^0} & \dfrac{\partial \overline{T}^t}{\partial C_T^0} \\ \dfrac{\partial \eta^t}{\partial \overline{T}^0} & \dfrac{\partial \eta^t}{\partial \eta^0} & \dfrac{\partial \eta^t}{\partial T_b^0} & \dfrac{\partial \eta^t}{\partial C_T^0} \\ \dfrac{\partial T_b^t}{\partial \overline{T}^0} & \dfrac{\partial T_b^t}{\partial \eta^0} & \dfrac{\partial T_b^t}{\partial T_b^0} & \dfrac{\partial T_b^t}{\partial C_T^0} \\ \dfrac{\partial C_T^t}{\partial \overline{T}^0} & \dfrac{\partial C_T^t}{\partial \eta^0} & \dfrac{\partial C_T^t}{\partial T_b^0} & \dfrac{\partial C_T^t}{\partial C_T^0} \end{bmatrix}$$

stratified/mixed regime

$$\mathbf{M} = \left[\dfrac{\partial \overline{T}^t}{\partial \overline{T}^0} \right]$$

$$\mathbf{M} = \frac{\partial x_i^t}{\partial x_j^0}$$ is calculated numerically: from the model runs with perturbed initial values.



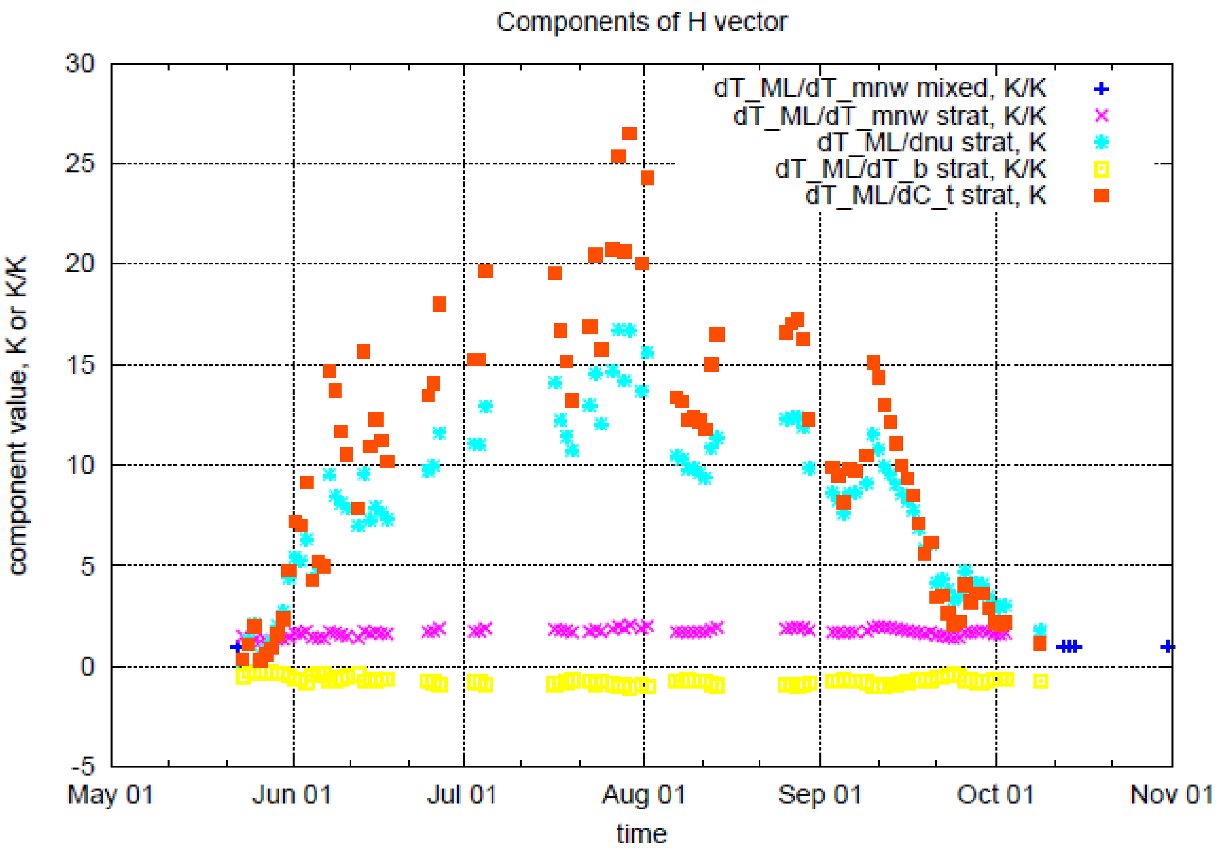
Perturbations

stratified/mixed regime

$$\delta \mathbf{X}^0 = \begin{bmatrix} 0.2K & 0.05 & 0.1K & 0.05 \end{bmatrix}^{\mathrm{T}} \qquad \delta \mathbf{X}^0 = \begin{bmatrix} 0.2K \end{bmatrix}$$

Components of vector H (sensitivity)

Lake Inarijärvi, 14.3 m



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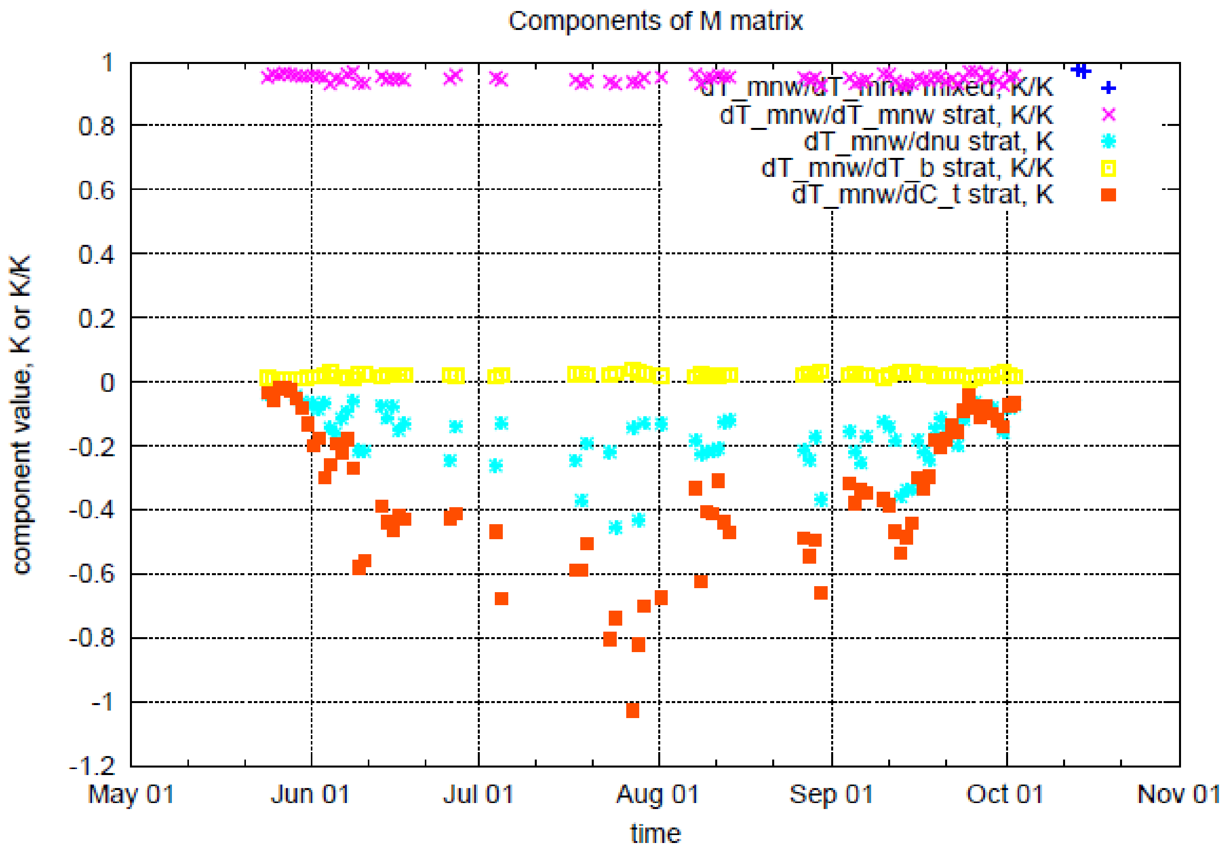
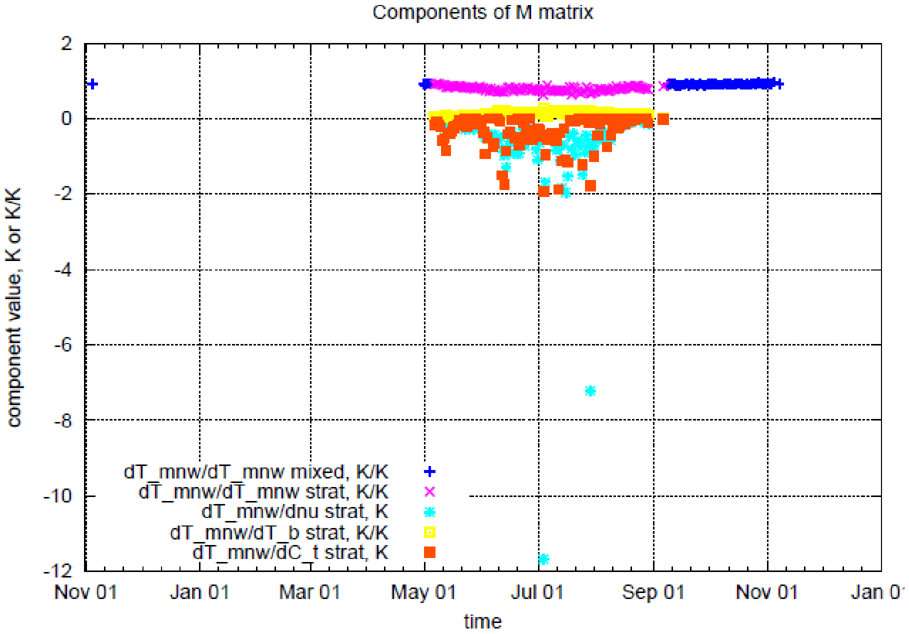
Matrix M, components of $\bar{T}$ (sensitivity)

Lake Inarijärvi, 14.3 m

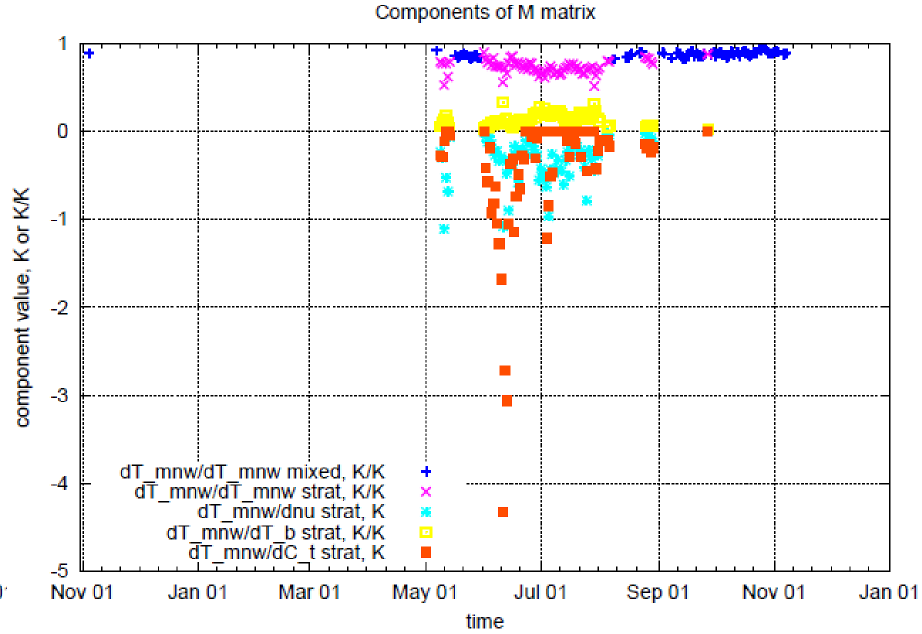


Matrix M, components of $\bar{T}$ (sensitivity)

Lake Jääsjärvi, 4.6 m

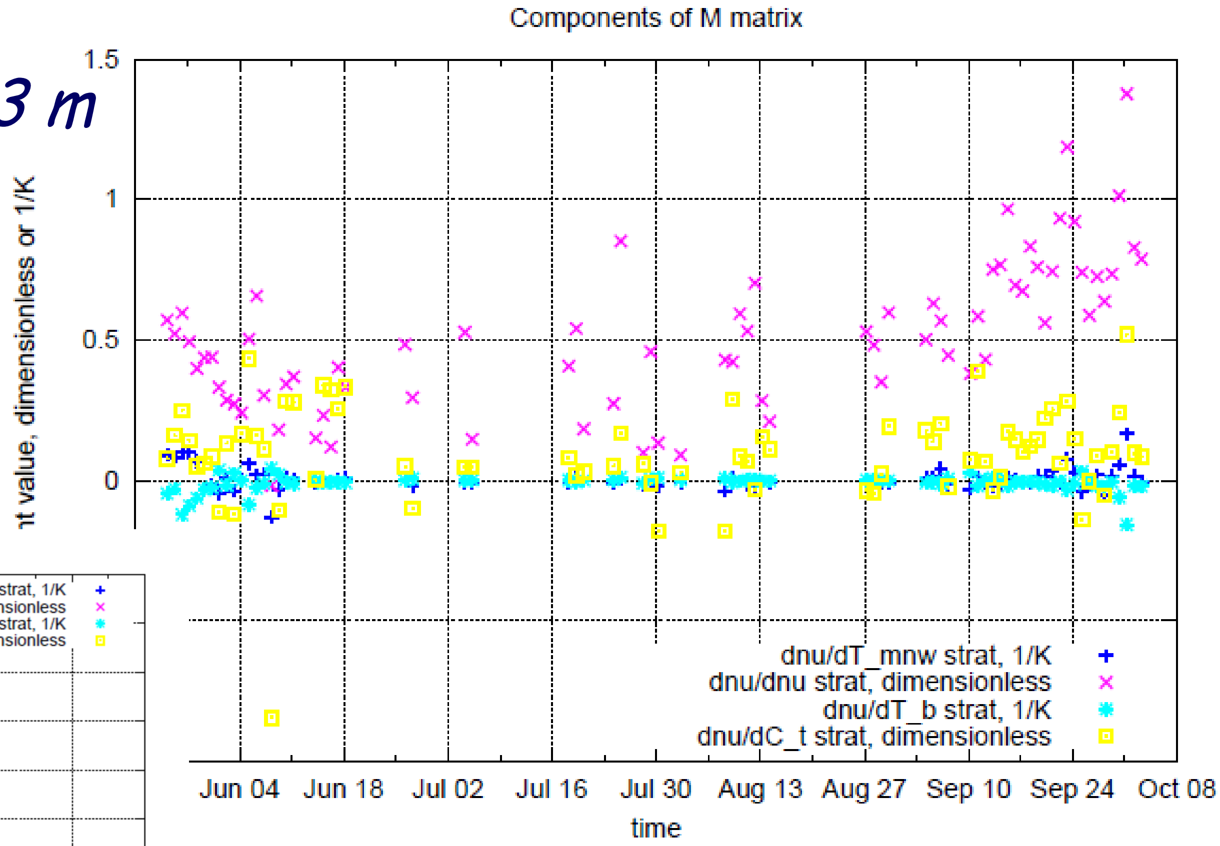


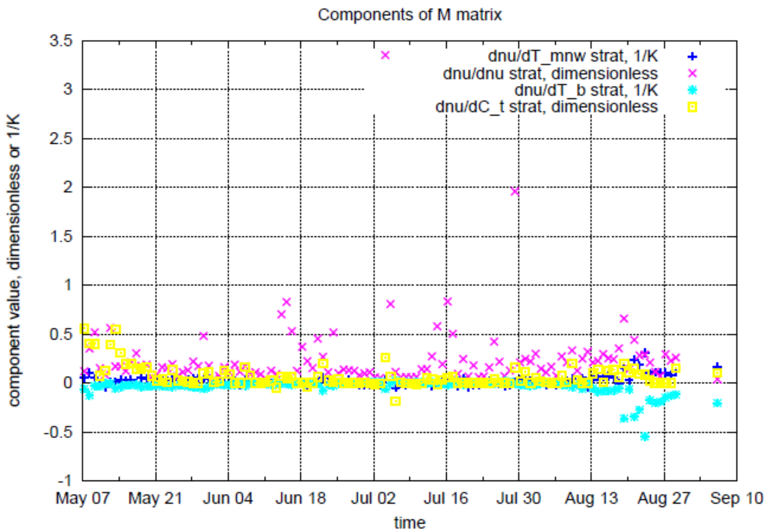
Lake Tuusulanjärvi, 3.2 m



Matrix M, components of η (sensitivity)

Lake Inarijärvi, 14.3 m





Lake Jääsjärvi, 4.6 m

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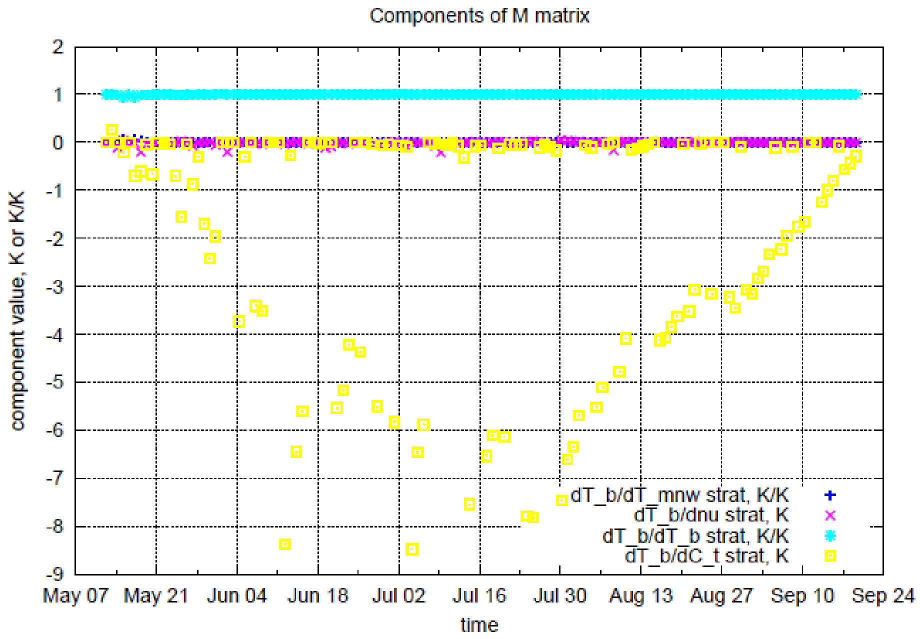
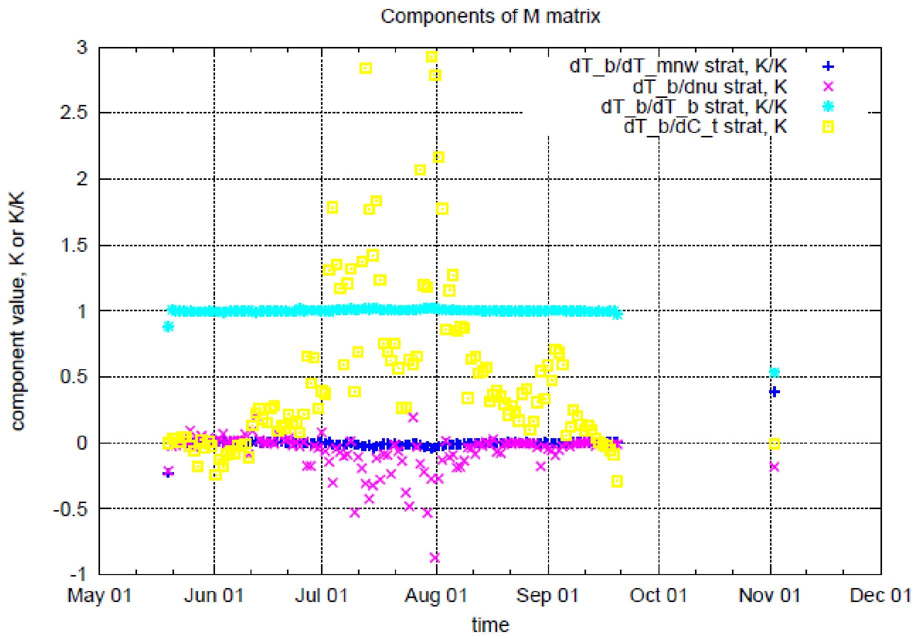
Matrix M, components of T_b (sensitivity)

Lake Kallavesi, 9.7 m

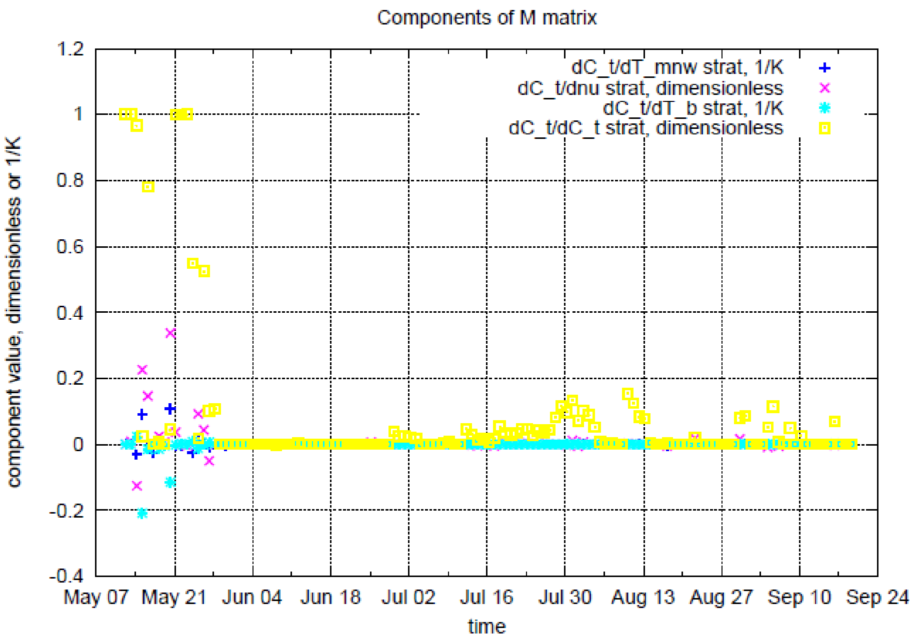


Lake Rehjä-Nuusjärvi, 8.5 m

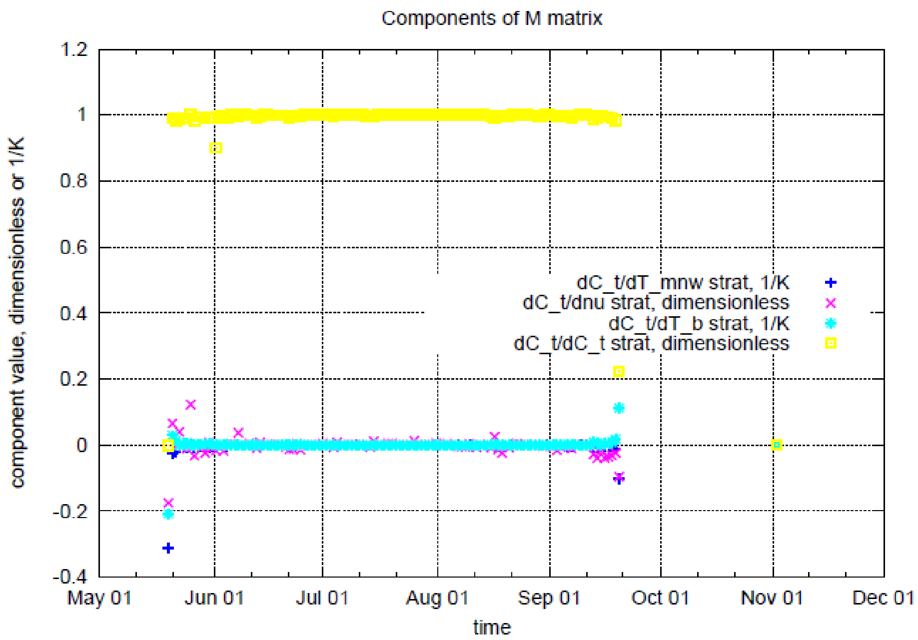


Matrix M, components of C_T (sensitivity)

Lake Kallavesi, 9.7 m



Lake Rehjä-Nuusjärvi, 8.5 m



Obs error variance: $\mathbf{R} = 1.0\,\mathrm{K}^2$

Model error variances and co-variances:

stratified regime

$$\mathbf{Q} = \begin{bmatrix} 4.000K^2 & 0.100K & -1.000K^2 & -0.100K \\ 0.100K & 0.010 & -0.050K & 0.005 \\ -1.000K^2 & -0.050K & 1.000K^2 & 0.050K \\ -0.100K & 0.005 & 0.050K & 0.010 \end{bmatrix}$$

mixed regime

$$\mathbf{Q} = \left[4.0K^2\right]$$

$\mathbf{x}_B$ - *background vector*

$\mathbf{x}_A$ - *analysis vector*

$\mathbf{B}$ - *background error covariance matrix*

$\mathbf{A}$ - *analysis error covariance matrix*

$\mathbf{K}$ - *Kalman gain vector (weights)*

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$$\mathbf{x}_B = M(\mathbf{x}_A)$$

$$\mathbf{B} = \mathbf{M}\mathbf{A}\mathbf{M}^{\mathrm{T}} + \mathbf{Q}$$

$$\mathbf{K} = \mathbf{B}\mathbf{H}^{\mathrm{T}}\left(\mathbf{H}\mathbf{B}\mathbf{H}^{\mathrm{T}} + \mathbf{R}\right)^{-1}$$

$$\mathbf{x}_{\mathrm{A}} = \mathbf{x}_{\mathrm{B}} + \mathbf{K}\left(\mathbf{y} - H\left(\mathbf{x}_{\mathrm{B}}\right)\right)$$

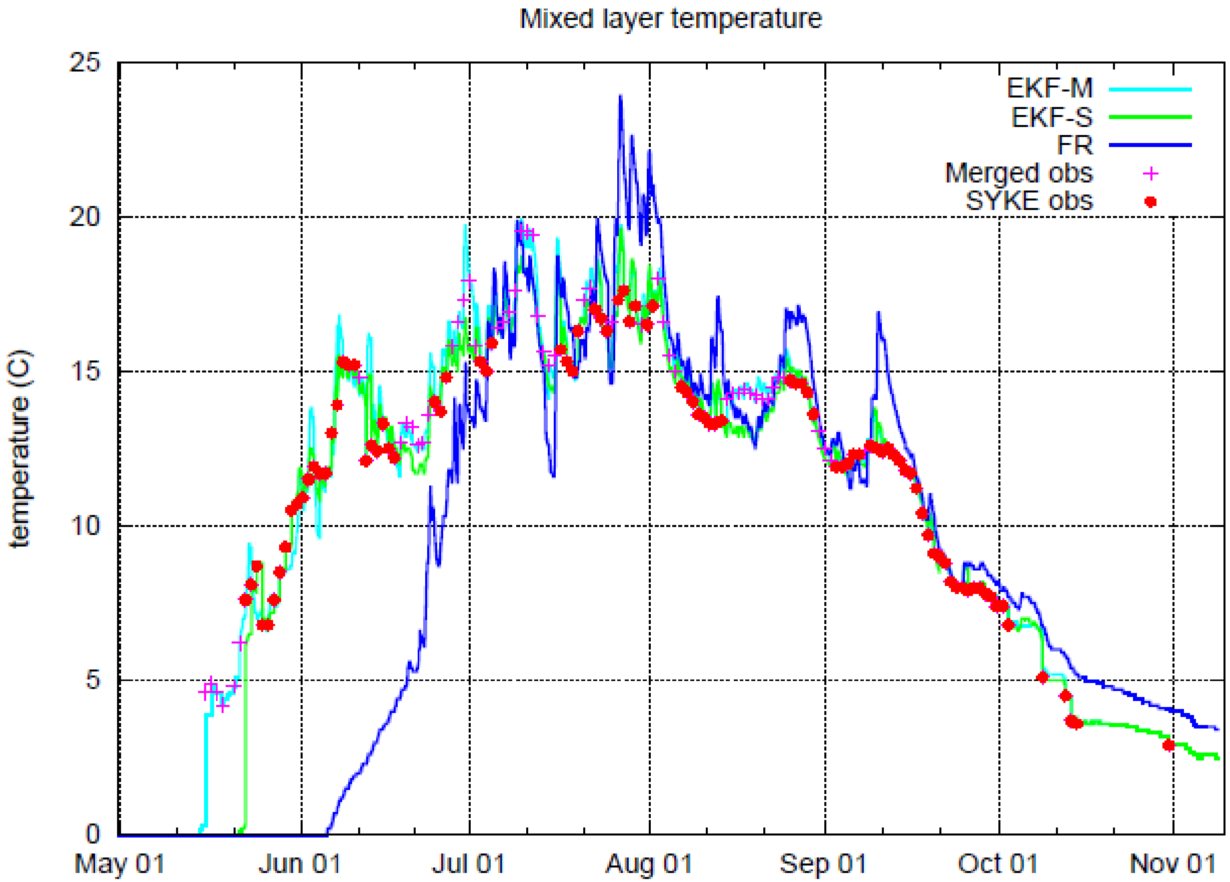
$$\mathbf{A} = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{B}$$

Assimilation experiments

- 27 lakes with SYKE obs,
  among them 4 lakes with merged SYKE+MODIS obs
- 3.11.2010-10.11.2011
- FLake offline
- Forcing from operational HIRLAM forecasts
- Obs at 8.00 UTC
- Analysis at 6.00 UTC
- Assimilation window - 2h
- Assimilation cycle - 24 hours

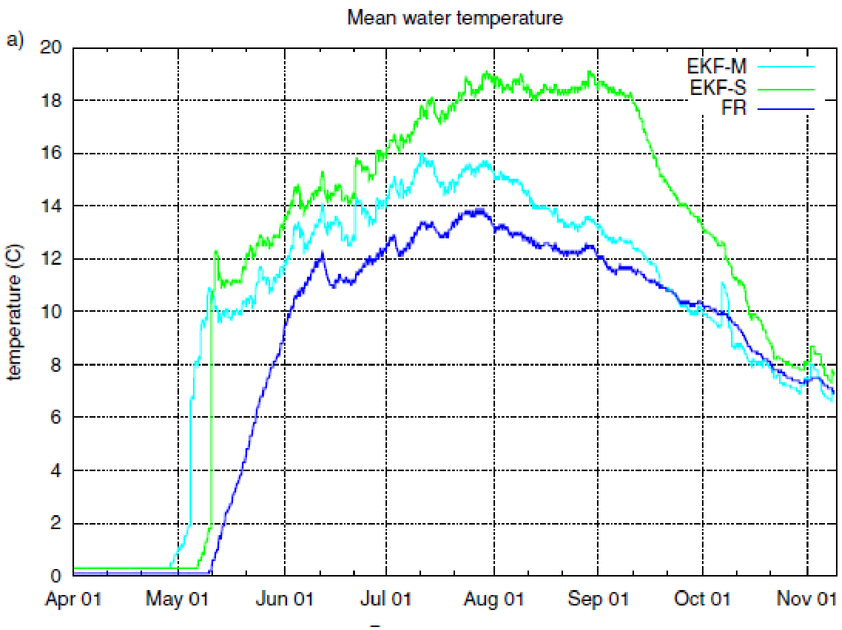
Simulations of T_{ML}

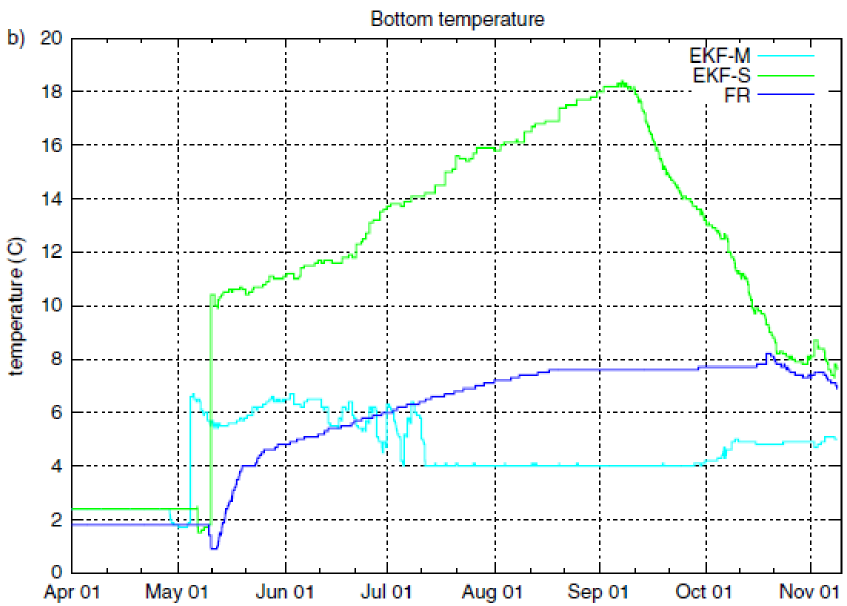
Lake Inarijärvi, 14.3 m



Simulations of $\overline{T}$ and T_b

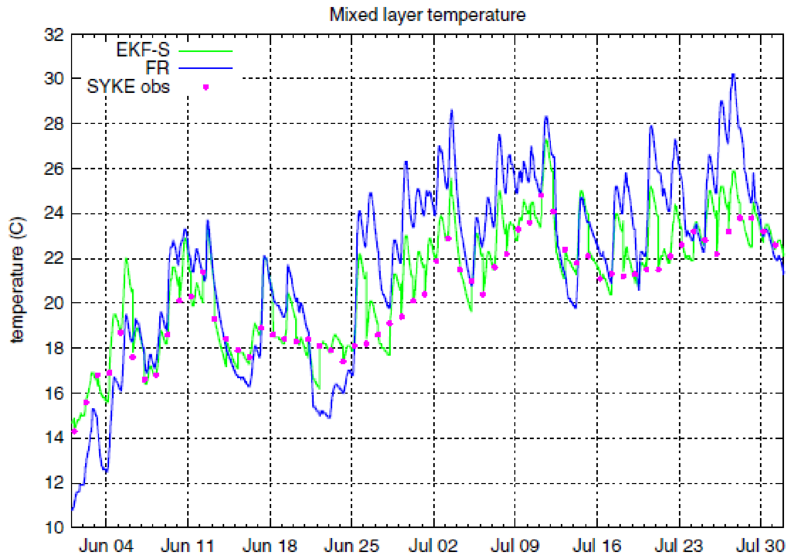
Lake Saimaa, 10.8 m

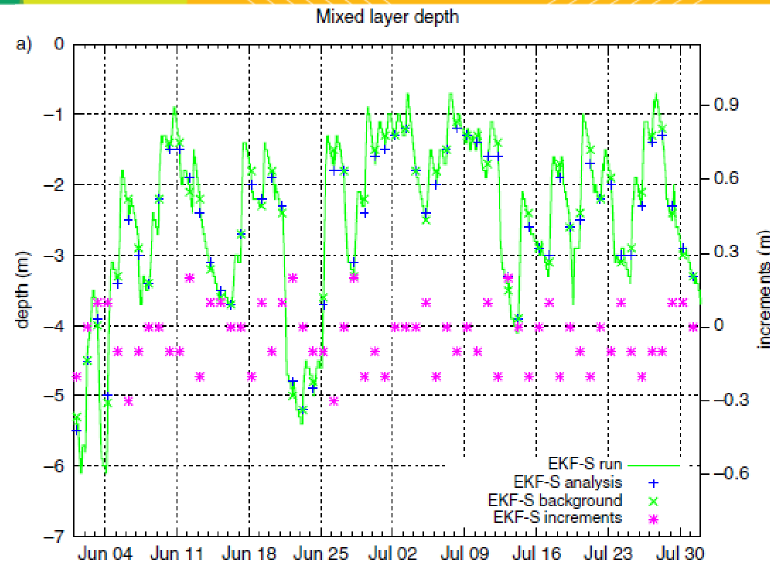


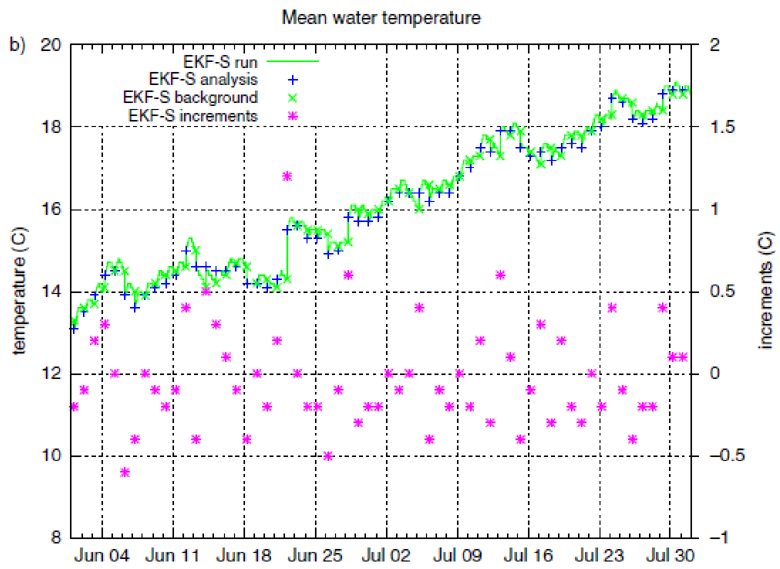


Decoupling problem

Lake Saimaa, 10.8 m







Bias in h ? + decoupling

Impact of obs:

$$I = \frac{RMSE_{\mathrm{mod}} - RMSE_{assim}}{RMSE_{\mathrm{mod}}} \cdot 100\%$$

Name (lon, lat)	D ,m	I, %	Name (lon, lat)	D ,m	I, %
Kuivajärvi (23.9,60.8)	2.2	94.8	Rehja-Nuasjärvi (28.0,64.2)	8.5	95.5
Tuusulanjärvi (25.1,60.4)	3.2	94.3	Vaskivesi (23.8,62.1)	7.0	97.1
Pääjärvi 1 (24.5,62.9)	3.8	96.6	Haukivesi (28.4,62.1)	9.1	94.9
Pesiöjärvi (28.7,64.9)	3.9	95.4	Kallavesi (27.7,62.8)	9.7	96.3
Kyyvesi (27.1,62.0)	4.4	96.5	Pielinen (29.6,63.3)	10.1	94.6
Jääsjärvi (26.1,61.6)	4.6	96.2	Konnevesi (26.6,62.6)	10.6	95.4
Nilakka (26.5,63.1)	4.9	96.6	Saimaa (28.1,61.3)	10.8	94.5
Pyhäjärvi (22.3,61.0)	5.5	96.4	Ala-Rieveli (26.2,61.3)	11.2	92.4
Längelmävesi (24.4,61.5)	6.8	94.4	Päijänne (25.5,61.6)	14.1	93.7
Ounasjärvi (23.6,68.4)	6.6	97.3	Inarijärvi (27.9,69.1)	14.3	97.1
Lappajärvi (23.7,63.1)	6.9	93.4	Näsijärvi (23.8,61.6)	14.7	94.0
Oulujärvi (27.0,64.5)	7.0	95.0	Pääjärvi 2 (25.1,61.1)	14.8	96.7
Unari (25.7,67.1)	7.0	94.0	Kilpisjärvi (20.8,69.0)	19.5	96.8
Kevojärvi (27.0,69.8)	7.0	98.0			

Cross-validation

Every second obs was assimilated, others were used for validation

Lake	no DA			EKF		
	Bias	RMSE	ESTD	Bias	RMSE	ESTD
Inarijärvi	-2.03	5.02	4.59	0.12	0.96	0.96
Saimaa	-1.07	3.67	3.52	-0.04	1.11	1.11
Lappajärvi	0.19	2.87	2.87	0.46	1.51	1.44
Tuusulanjärvi	0.83	2.92	2.80	0.85	1.41	1.13

Conclusions, perspectives:

- EKF algorithm to assimilate LWST in FLake gives promising results
- Components of vector H and matrix M evolve smoothly, show annual cycle: potential for simplifications
- Early spring observations are important, they may affect results on the deep water temperatures in summer
- Better specification of Q matrix (need observations of water temperature profiles)
- Study other components of EKF (B matrix, K vector), more a posteriori statistics
- Model bias corrections …
- Implementation … into SURFEX … HARMONIE

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Thank you for attention!

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